APPENDIX 5-4: Temperature Lapse Rate

The atmosphere is a complicated system of components which interact with one other in physically predictable ways. To understand the interaction, simplifying assumptions are necessary. One such simplifying assumption is to impose an adiabatic condition: given a "parcel" of air with some pressure (*P*), temperature (*T*), volume (*V*), and water content, it must move without exchanging heat with its environment. The sticking point here is that water in the parcel can exist in any of its three states, and can change among them. The processes of evaporation, condensation, fusion, and sublimation either require or give up heat, but these cannot occur under the adiabatic restriction. Therefore, it is further assumed that any water in the parcel must remain in the vapor phase, limiting the following discussion to "dry" air only. That is, since water vapor-laden air is lighter than air without moisture, a parcel will be buoyant according to Archimedes' principal and will naturally rise. Following the upward motion of such a parcel will reveal its temperature change with altitude.

Applying the First Law of Thermodynamics,[*]

$$dE = dQ + dW \quad , \tag{1}$$

to an air parcel, the adiabatic assumption (no heat transfer, $dQ = 0$) leaves

$$dE = dW \tag{2}$$

If a parcel expands as it rises, it will do work on its environment,

$$dW = -PdV \tag{3}$$

This is at the expense of its internal energy:

$$dE = \nu C_V dT \quad , \tag{4}$$

where ν is the number of moles and C_V is the molar heat capacity at constant volume. Thus

$$\nu C_V dT = -PdV \quad , \tag{5}$$

which says that an increase in volume must cause a corresponding decrease in temperature. This is exactly what is observed: air gets colder as it climbs to higher altitudes.[†] This can be expressed as a "lapse rate," $\frac{dT}{dz}$.

Using the state equation for an ideal gas in the form

$$PV = \nu RT \quad , \tag{6}$$

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[*] The amount of energy in a small air parcel, *dE*, is a combination of its internal heat content, *dQ*, and the "work" done to compress it, *dW*. But neither heat nor work are "exact differentials," in that they can occur in several different ways, just so long as the sum of their changes adds up to the change in a parcel's energy.

[†] Consider a vacation driving into the mountains.

differentiate it explicitly as

$$PdV + VdP = \nu RdT \tag{7}$$

This can be substituted into the energy equation and like terms can be collected to obtain

$$\nu\left(C_V + R\right)dT \equiv \nu C_P dT = VdP \tag{8}$$

where C_P is the molar heat capacity at constant pressure. Convert the molar heat capacity to specific (per unit mass) heat capacity, and express the number of moles by using *A*, the "atomic weight" (mass per mole) of air:

$$C_P = c_P A \text{ and } \nu = {}^{M}\!/_{A} \tag{9}$$

where *M* is the mass of the air parcel. Making these substitutions,

$$Mc_P dT = VdP \tag{10}$$

Picking up the hydrostatic condition,

$$dP = -n\langle m\rangle g\,dz = -\frac{M}{V}g\,dz\,, \tag{11}$$

and placing it into the last expression,

$$Mc_P dT = -V\frac{M}{V}g\,dz = -Mg\,dz \tag{12}$$

Finally dividing by the parcel's mass, the result is

$$\boxed{\frac{dT}{dz} = -\frac{g}{c_P}} \tag{13}$$

A quick internet search finds that the specific heat at constant pressure for what is called tropospheric dry air is about 1.01 kJ·kg^{-1}K^{-1}. This gives a lapse rate of about –9.7 K·km^{-1}.

Presumably, all of the air parcels in a given region have a common relative humidity. As they rise, adiabatically cooling according to the lapse rate we've calculated, they will all reach saturation at about the same altitude. This is recognized as being the bottoms of the clouds. Moist air continuing to rise will experience an internal energy input of the released latent heat of condensation from the water droplets forming. Hence the lapse rate will decrease to about half of its dry value.