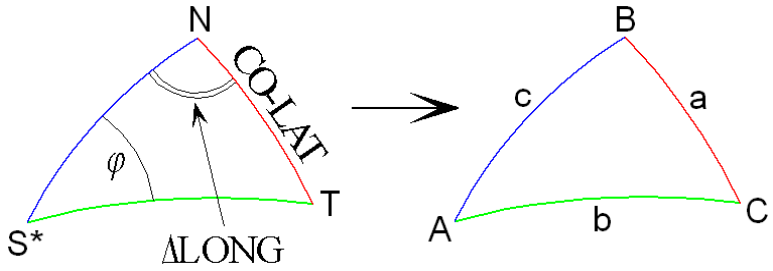
APPENDIX 4-4
Spherical Earth Geolocation Calculation

First re-label the spherical triangle for convenience of notation:



GIVEN: two sides and the included angle, A, b, c

FIND: remaining side and angles, a, B, C

SOLUTION:

1. Use *Napier's analogies* to find the sum and difference of angles B and C:

$$B + C = 2\tan^{-1}\left[\frac{\cos\frac{1}{2}(b-c)}{\cos\frac{1}{2}(b+c)}\cot\tfrac{1}{2}A\right] = \mathbf{X}$$

$$B - C = 2\tan^{-1}\left[\frac{\sin\frac{1}{2}(b-c)}{\sin\frac{1}{2}(b+c)}\cot\tfrac{1}{2}A\right] = \mathbf{Y}$$

2. Solve for angles B and C:

$$B = \frac{\mathbf{X}+\mathbf{Y}}{2} \quad \text{and} \quad C = \frac{\mathbf{X}-\mathbf{Y}}{2}$$

3. Use Law of Sines to find side a

$$A = \sin^{-1}\left[\frac{\sin A \sin b}{\sin B}\right] = \sin^{-1}\left[\frac{\sin A \sin c}{\sin C}\right]$$

The geolocation of T is therefore

$$\boxed{\begin{aligned} LAT[T] &= 90° - a \\ LONG[T] &= LONG[S^*] + B \end{aligned}}$$

A note of caution however: the sensor and target may be on opposite sides of the Prime Meridian or the Date Line, so some care is required to get the target into the correct hemisphere.