# SUGGESTED SOLUTIONS (ODD)

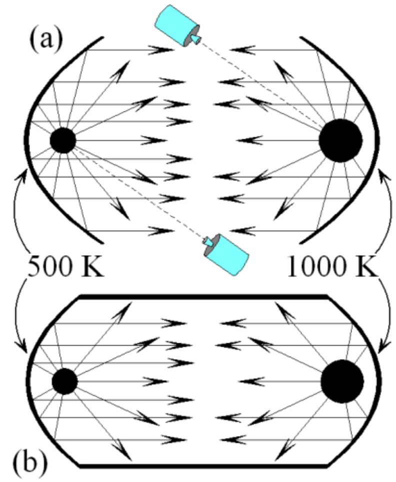
## CHAPTER 3

NOTE: Use three-digit precision for all calculations unless otherwise stated or implied.

**3-1.** ***Speculation on Emissivity and Reflectivity.***

(a) In a laboratory, two blackbody sources are set up facing each other. One blackbody is at 500 K and the other is at 1000 K. External sensors are set up to measure their spectra. With light from the 500 K blackbody shining on the 1000 K blackbody, and vice versa, what spectra do the sensors see?

(b) If the 500 K and 1000 K blackbodies were thermally isolated from the world (they are the only objects in their "universe"), what would be their emission spectra? (This is a theoretical question because, of course, we have no way of actually measuring the spectra without interacting with the objects. That is, we don't exist in their universe.)




SUGGESTED DISCUSSION:

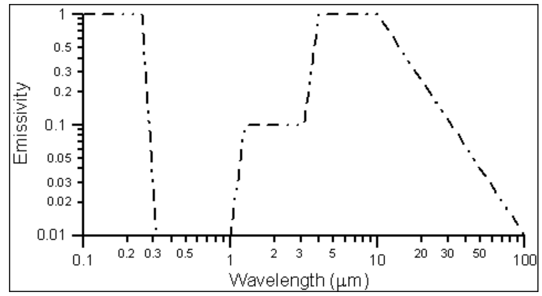
(a) This question tests your faith in the definition of a blackbody. If we claim that an object is a blackbody, then we are asserting that its temperature is constant (forever and ever), it is in steady state with its surroundings, and its emitted spectrum (emissivity = 1) is the Planck function. If these conditions are indeed satisfied, then it doesn't matter what the "temperature" of the incident light is from any external sources falling on a blackbody – it will absorb it all and will re-emit it with the spectrum characteristic of its own temperature. So the 1000 K object or the 500 K object will emit blackbody radiation characteristic of 1000 K and 500 K perfect emitters, respectively. (However, since there is not enough environmental radiation for the two blackbodies to maintain their temperatures, there must be some other source of energy powering them ~ like electrical heaters.)

(b) In this case, there is only a certain amount of energy contained within the objects' universe. That energy will eventually become shared between the two objects such that they will come to the same temperature ~ the 500 K object heating up and the 1000 K object cooling down. What that final temperature is depends on the masses and thermal capacities of the objects. Of course, this action negates the initial assumption that the two objects are blackbodies. True blackbodies never change temperature.

**3-3.** ***Reflectivity and Emissivity.***

(a) For the alien material whose emissivity is shown at right (from Problem 2-23), calculate and plot its spectral reflectivity.

(b) If an external 2727ºC blackbody source shines on a sample of this material, calculate and plot the reflected spectral exitance.




☞ See spreadsheet Chapter 03 ~ Suggested Solutions (Odd).xlsx. ☜

**3-5.** ***Reflectivity and Emissivity.*** In the February 2004 issue of *The Physics Teacher*, Prof. Paul Hewitt proposed the following problem in the monthly "Figuring Physics" column. (You should contrast this with Problem 2-18.)

You're a consultant to a manufacturer of space gear that wants to encase some instruments in a covering that will have two properties: (1) absorb as little energy as possible on the side of the package facing the sun, and (2) emit as little energy as possible on the side away from the sun. You should recommend a covering with …

(a) the side facing the sun is black and the other side is shiny
(b) the side facing the sun is shiny and the other side is black
(c) both sides are shiny
(d) both sides are black

SUGGESTED ANSWER: Prof. Hewitt's answer in the next issue of *The Physics Teacher* is as follows: Surfaces that absorb more light (and usually more infrared radiation as well) are black. So you don't want the side in the sun to be black. Go for shiny, which will reflect rather than absorb energy. Since poor absorbers are also poor emitters, go for shiny on the side facing away from the sun also. So shiny surfaces on both sides of the covering will (answer c) mean less absorption of solar radiation on one side and less emission on the other side. (Light incident upon a surface can be either absorbed or reflected. Usually both occur to some degree. When absorption dominates, the surface is black. When reflection dominates, the surface is shiny. The rate of absorption or reflection depends on surface composition – the *emissivity* of the object's surface.)

Is the official answer surprising to you? It shouldn't be if you think of all the pictures you've ever seen of satellites all covered with what looks like foil. (Contrast this to Problem 2-18.) Another related question that you might think about is: what color would you want to paint your house if you lived in Alaska – white or black? (What color are igloos and polar bears?)

**3-7.** ***Polarized Reflection.*** You are driving directly into the sun, which is 30° above the horizon. There is a tremendous glare off the hood of your car. (Assume your hood is horizontal.) If the index of refraction of the paint on your hood is approximately 2.75, what is the degree of polarization of the sunlight reflecting into your eyes? What would the brightness of the glare be if you were wearing polarizing glasses (relative to not wearing polarizing glasses)?

SUGGESTED SOLUTION: Since we assume that the hood of our car is horizontal (this is a very old-fashioned car), we note that if the sun is 30° above the horizon, the angle of incidence is 60°, measured from the normal. Using Snell's Law, we then calculate the transmitting angle ~

$$\sin\theta_t = \frac{n_{AIR}}{n_{PAINT}}\sin\theta_i = \frac{1.00}{2.75}\sin 60^\circ \quad\Rightarrow\quad \theta_t \approx 18.36^\circ$$

Calculating the reflectances of the p- and s-waves, we have ~

$$R_{TM} = \left(\frac{-n_t\cos\theta_i + n_i\cos\theta_t}{n_t\cos\theta_i + n_i\cos\theta_t}\right)^2 = \left(\frac{-2.75\cos 60^\circ + \cos 18.36^\circ}{2.75\cos 60^\circ + \cos 18.36^\circ}\right)^2 \approx 0.0336 \text{ and}$$

$$R_{TE} = \left(\frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}\right)^2 = \left(\frac{\cos 60^\circ - 2.75\cos 18.36^\circ}{\cos 60^\circ + 2.75\cos 18.36^\circ}\right)^2 \approx 0.4603$$

The degree of polarization is therefore ~

$$P = \frac{R_{TE} - R_{TM}}{R_{TE} + R_{TM}} = \frac{0.4603 - 0.0336}{0.4603 + 0.0336} \approx 0.864$$

Evidently, if you don't wear your polaroids, you will see ≈ 46% + 3.4% ≈ 49.4% of the sunlight glaring off the hood of your car. If you do wear your polaroids, you will only see 3.4%. (The reflection appears to be about 14.5 times "brighter" by not wearing polarizing sunglasses).

☞ See Chapter 03 ~ Suggested Solutions (Odd).xlsx for an interactive spreadsheet of the Fresnel equation plots. ☜

QUESTION: If 49.4% of the incident sunlight is reflected, where does the other 50.6% go?

ANSWER: It is "transmitted" into the paint on your hood and absorbed. So what does the paint on the hood of your car do with the energy? Of the three modes for moving energy around, the paint *conducts* some of the energy to its metal substrate, it *convects* energy into the air right above it, and it *radiates*. If you put your hand on the hood, you will experience the first mode. (Does this explain why you can fry an egg on the hood of your car on a hot summer's day?); if you look across your hood at a shallow angle you may see a "mirage" which is the second mode; and of course you could observe the radiation with a thermal camera.

**3-9.** ***Normal Reflectivity.*** If we note that 10% of our light beam reflects back at us when we shine a flashlight on a certain dielectric material, what is the polarizing angle for light reflecting off the same material, and what fraction of the light is transmitted into the material at that angle?

SUGGESTED SOLUTION: If $R_\perp = 10\%$, then the index of refraction of the dielectric material is (assuming air is the other medium) ~

$$R_\perp = \left(\frac{n_t - n_i}{n_t + n_i}\right)^2 \quad \Rightarrow \quad n_t = \frac{1+\sqrt{R_\perp}}{1-\sqrt{R_\perp}} n_i \approx 1.92$$

The polarization angle is then ~

$$\theta_P = \tan^{-1}\frac{n_{DIELECTRIC}}{n_{AIR}} = \tan^{-1}1.92 \approx 62.5°,$$

and the "transmitted" angle, from Snell's Law is ~

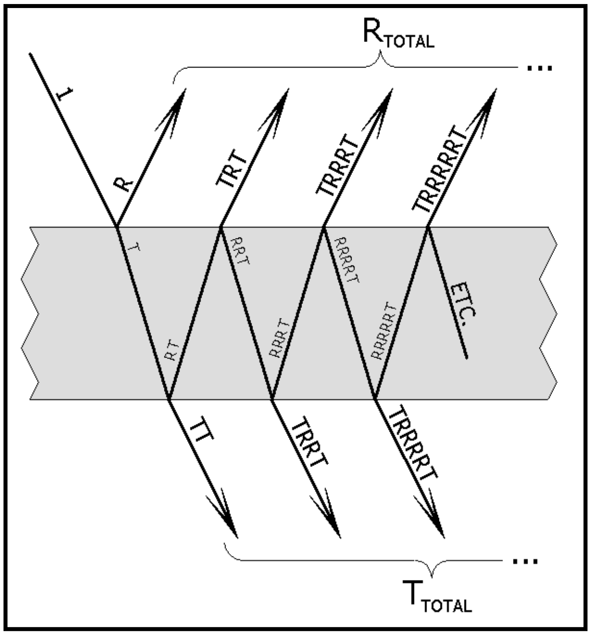
$$\theta_t = \sin^{-1}\left(\frac{n_{AIR}}{n_t}\sin\theta_P\right) \quad \Rightarrow \quad \theta_t \approx 27.5°$$

At the polarization angle of incidence, $R_{TM} = 0$, of course, but we still have reflected $R_{TE}$ ~

$$R_{TE} = \left(\frac{n_i\cos\theta_i - n_t\cos\theta_t}{n_i\cos\theta_i + n_t\cos\theta_t}\right)^2 = \left(\frac{\cos 62.5° - 1.92\cos 27.5°}{\cos 62.5° + 1.92\cos 27.5°}\right)^2 \approx 0.329$$

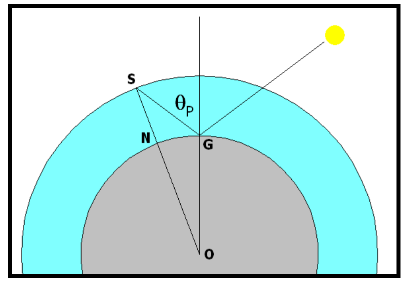
The amount transmitted into the medium is therefore 67.1%.

But wait, maybe there's more! We only considered reflection from a single surface. What happens to the fraction that's transmitted into the medium if it encounters another interface between materials? It will partially reflect/transmit again, and the fraction that is reflected will return to the original surface. What happens to it there? It will reflect/transmit again … and again … and again … and again … The drawing below suggests what is going on, and you can actually observe this if you shine your laser at a pane of glass at a slight angle.




So in our initial calculation (10% of our flashlight reflects back to us) shouldn't we account for all of these internal reflections? Of course we should, and the answer looks like it would be some kind of infinite series that requires a clever mathematician to figure out. Fortunately, this has been done for us, but we'll not reveal the answer at this time: we'll save it for the chapter on spectral filters (Chapter 9). The picture we've sketched at right is the basis of the "thin film" interference filter that actually does more than just reflect and transmit light as shown here – it also selects light of different wavelengths to reflect and transmit. That is because the thickness of the layer must also be considered.

**3-11.** ***Sun Glint.*** The sun's reflection off the Earth's surface is generally called "glint." A sensor on a satellite at an altitude of 830 km above the surface of the Earth sees the glint off the Pacific Ocean to be 100% polarized. At what angle is the sensor pointing above its nadir? (Nadir is defined as the direction straight "down" toward the center of the Earth.) [The index of refraction of salt water is approximately 1.35, and the radius of the Earth is 6370 km.]




SUGGESTED SOLUTION: The polarization angle,

$$\theta_P = \tan^{-1}\frac{n_{WATER}}{n_{AIR}} = \tan^{-1}1.35 \approx 53.5°,$$

is an exterior angle of $\Delta OGS$, so $\angle OGS \approx 126.5°$.
We can thus use the Sine Law ~

$$\frac{\sin\angle OSG}{\overline{OG}} = \frac{\sin\angle OGS}{\overline{OS}}$$

$$\sin\angle OSG = \frac{\overline{OG}}{\overline{OS}}\sin\angle OGS$$

$$\angle OSG = \sin^{-1}\left(\frac{6370\,\text{km}}{(6370\,\text{km} + 830\,\text{km}}\sin 126.5°\right) \approx 45.3°$$