

APPENDIX 3-1: Lambertian Reflectivity Phase Function

To show that all reflected light is accounted for by the Lambertian reflectivity phase function, the integral in Equation 3-19 goes as follows ~

$$\begin{aligned}\int_{HEMISPHERE} \phi(\theta_r) d\Omega_R &= \int_0^{2\pi} \int_0^{\pi/2} \frac{\cos \theta}{\pi} \sin \theta_r d\theta_r d\varphi_r \\&= \frac{1}{\pi} \int_0^{2\pi} d\varphi_r \int_0^{\pi/2} \cos \theta_r \sin \theta_r d\theta_r \\&= \frac{1}{\pi} [\varphi_r]_0^{2\pi} \left[\frac{-\cos^2 \theta_r}{2} \right]_0^{\pi/2} \\&= \frac{1}{\pi} [2\pi - 0] \left[\frac{-\cos^2 \pi/2}{2} - \frac{-\cos^2 0}{2} \right] \\&= \frac{1}{\pi} [2\pi] \left[\frac{1}{2} \right] \\&= 1\end{aligned}$$