SUGGESTED SOLUTIONS (ODD)

CHAPTER 16

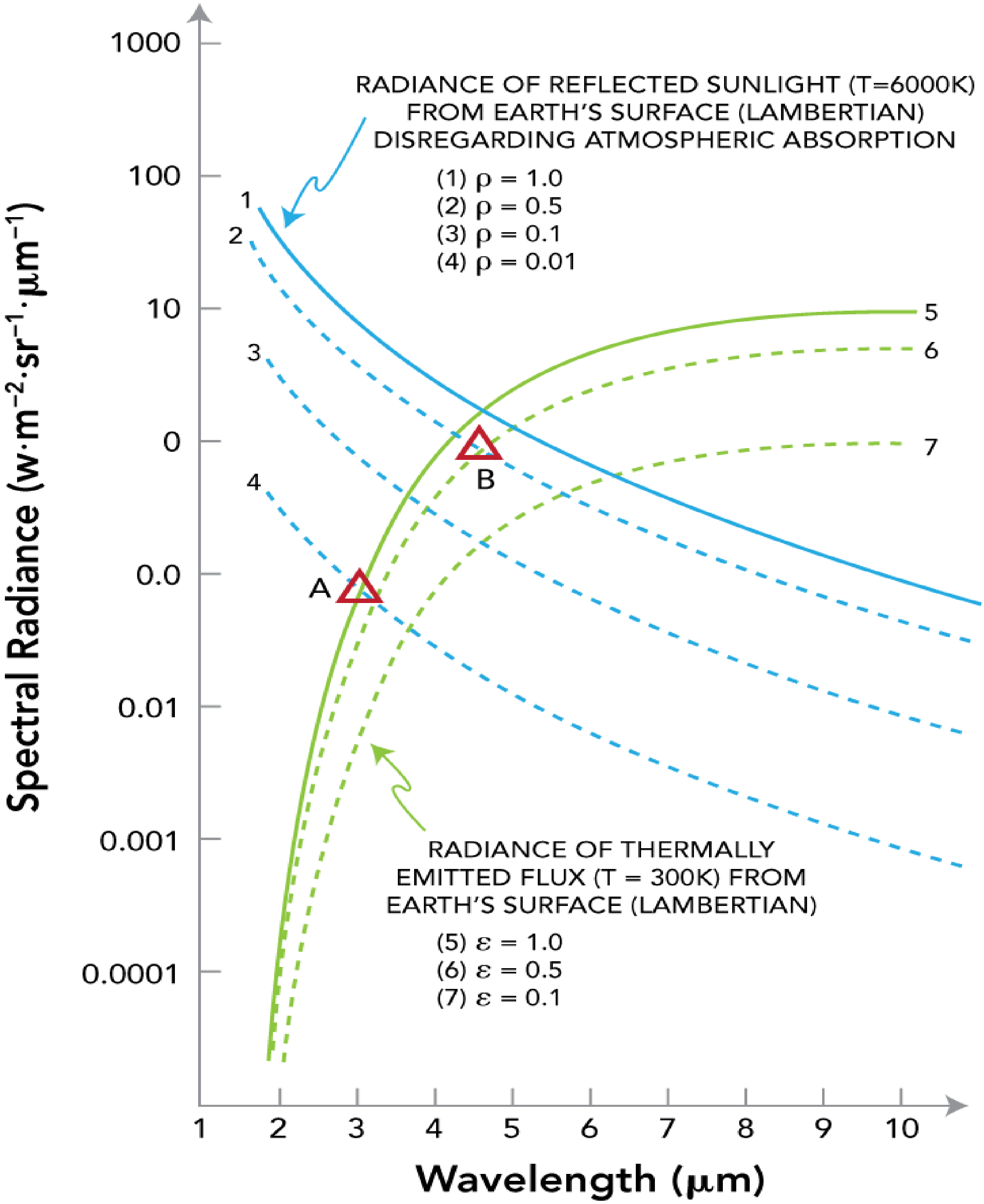
16-1 Consider the radiance values from reflected sunlight and from thermal emission in the attached chart. At what wavelengths are these two components equal, if the:

A. Reflectivity is 0.01 and emissivity is 0.99?

B. Reflectivity is 0.5 and emissivity is 0.5?

C. Reflectivity is 0.99 and emissivity is 0.01? Hint: If the appropriate curves are not shown on the chart, make an estimate from the other curves present.

D. From the above results, what can be concluded about the MWIR being referred to as part of the thermal (or emissive) portion of the EM spectrum during the day? During the night?



1000
100
10
0
0.0
0.01
0.001
0.0001
Spectral Radiance (w·m⁻²·sr⁻¹·µm⁻¹)
RADIANCE OF REFLECTED SUNLIGHT (T=6000K)
FROM EARTH'S SURFACE (LAMBERTIAN)
DISREGARDING ATMOSPHERIC ABSORPTION
(1) ρ = 1.0
(2) ρ = 0.5
(3) ρ = 0.1
(4) ρ = 0.01
1
2
3
4
5
6
7
A
B
RADIANCE OF THERMALLY
EMITTED FLUX (T = 300K) FROM
EARTH'S SURFACE (LAMBERTIAN)
(5) ε = 1.0
(6) ε = 0.5
(7) ε = 0.1
1 2 3 4 5 6 7 8 9 10
Wavelength (µm)

SUGGESTED SOLUTION:

A) Locate the point on the graph where the reflected sunlight curve for $\rho = 0.01$ (curve 4) intersects with the emitted earth surface thermal emission curve for $\varepsilon = 0.99$ (curve 5) where $\varepsilon = 1.0$, is close enough. This point (A on the graph) has $\lambda = 3.1$ μm coordinate.
(Comment: Water, vegetation and some soils have these values).

B) For $\rho = 0.5$ (curve 2) and $\varepsilon = 0.5$ (curve 6), the intersection point (now point B on graph) shifts to higher $\lambda = 4.7$ μm.
(Comment: Some soils, paints and dyes).

C) For $\rho = 0.9$ (curve 1) and $\varepsilon = 0.1$ (curve 7), the intersection point is located at $\lambda = 6.1$ μm. So the $\rho = 0.99$, $\varepsilon = 0.01$ point would be at larger λ, perhaps $\lambda \cong 10$ μm.
(Comment: Metals and mirrors).

D) During the day, MWIR is a combination of solar reflective and object emissive, and the "boundary" between the two regions varies with ρ and ε values of various objects..
During the night, MWIR is totally emissive.

16-3. Most manmade objects have surfaces with average LWIR emissivity values in the LWIR that range from 0.8 to 1.0. Their average reflectivity values in the visible-SWIR range from 0.05 to 0.50. Assume that the area of the object pointed at the sun is 0.5 of the total radiating surface area. The solar irradiance (integrated over all λ) reaching the earth's surface at a particular location and day of the year is 800 watts / m^2. Ignore absorption of the down-welling radiance from the sky (atmosphere.)

A. What is the maximum value possible for the steady state temperature of the object under these conditions?

B. What is the minimum value possible for its steady state temperature?

C. Based on the results above, do you think that absorption of the down-welling radiance from the sky (atmosphere) can be significant?

SUGGESTED SOLUTION:

Steady-state temperature in the absence of down-welling radiance

$$T_{ss} = \left(\frac{\alpha_{vis}\ A_\rho\ E_{sun}}{\varepsilon_{LW}\ A_s\ \sigma} \right)^{1/4}$$

First consider

$$\left(\frac{A_\rho\ E_{sun}}{A_s\ \sigma} \right)^{1/4} = \left[\left(\frac{1}{2} \right) \left(\frac{800\ \cancel{\text{watts}} / \cancel{m^2}}{5.67 \times 10^{-8}\ \cancel{\text{watts}} / \cancel{m^2}\ K^4} \right) \right]^{1/4}$$

$= 289.8$ K

A) T_{ss} is a max when α_{vis} is largest and ε_{LW} is smallest

$$
\begin{aligned}
\left.T_{ss}\right)_{max} &= \left(\frac{0.95}{0.80}\right)^{1/4} (289.8\text{K}) \\
&= 1.044\ (289.8\text{K}) \\
&= 302.5\ \text{K} \quad (29^{o}\text{C or } 84^{o}\text{F})
\end{aligned}
$$

B) T_{ss} is a min when α_{vis} is smallest and ε_{LW} is largest

$$
\begin{aligned}
\left.T_{ss}\right)_{min} &= \left(\frac{0.5}{1.00}\right)^{1/4} (289.8\text{K}) \\
&= (0.841)\ (289.8\text{K}) \\
&= 243.7\text{K} \quad (-29^{o}\text{C or } -20.2^{o}\text{F})
\end{aligned}
$$

C) The results in part A) are low. Objects with α_{VIS} near one in value, for example asphalt, can attain a much higher daytime temperature. The results in part B) are low. Objects with α_{VIS} around 0.5, for example some forms of vegetation, can attain a much higher daytime temperature. The effects of object absorption of the atmospheric downwelling radiance are NOT negligible.

16-5. The mathematical approximations that θ(radians) = sin θ = tan θ and cos θ =1.0 work quite well when θ << 1 radian, and even when θ is somewhat larger. Complete the matrix below:

θ **(degrees)**	θ **(radians)**	**sin** θ	**tan** θ	**cos** θ
0				
5				
10				
15				
20				
25				
30				

Over what range of angles is each of the three approximations above valid to within 10 percent?

SUGGESTED SOLUTION:

θ (degrees)	θ (radians)	$\sin\theta$	$\tan\theta$	$\cos\theta$
0	0	0	0	1
5	0.087	0.087	0.087	0.996
10	0.175	0.174	0.176	0.985
15	0.262	0.259	0.268	0.966
20	0.349	0.342	0.364	0.94
25	0.436	0.423	0.466	0.906
30	0.524	0.5	0.577	0.866

$\theta > 30°$
$\theta = 43°$

$\theta < 30°$
$\theta = 25°$

$\theta < 30°$
$\theta = 29°$

Note: Recall the power series,

$$\sin\theta = \theta - \frac{\theta^3}{3!} + \frac{\theta^5}{5!} - \ldots$$

$$\cos\theta = 1 - \frac{\theta^2}{2!} + \frac{\theta^4}{4!} - \ldots$$

$$\tan\theta = \theta + \frac{\theta^3}{3} + \frac{2\theta^5}{15} + \ldots$$