SUGGESTED SOLUTIONS (ODD)

CHAPTER 6

NOTE: Use three-digit precision for all calculations unless otherwise stated or implied.

6-1. *Lens and Mirror Equations.* Show by algebraic manipulation that Gauss' formula for locating an image with a lens or mirror, $\frac{1}{d_{OBJ}} + \frac{1}{d_{IMG}} = \frac{1}{f}$, is the same as Newton's formula, $(d_{OBJ} - f)(d_{IMG} - f) = f^2$.

SUGGESTED SOLUTION: Starting with Gauss' formula, multiply through by $d_{OBJ} d_{IMG} f$ and cancel the common variables from numerator and denominator:

$$\frac{d_{OBJ} d_{IMG} f}{d_{OBJ}} + \frac{d_{OBJ} d_{IMG} f}{d_{IMG}} = \frac{d_{OBJ} d_{IMG} f}{f} \quad \Rightarrow \quad d_{IMG} f + d_{OBJ} f = d_{OBJ} d_{IMG} .$$

Rearrange into a more suggestive form:

$$d_{OBJ} d_{IMG} - d_{OBJ} f - d_{IMG} f = 0$$

The suggestion here is that this is starting to look like the high school algebra "FOIL" trick for multiplying two binomials – First-Outside-Inside-Last – which can be used in reverse for factoring. All that is missing is the "Last" term, which would be f^2. This is easily added to both sides of the equation, and the suggested factorization yields Newton's form:

$$d_{OBJ} d_{IMG} - d_{OBJ} f - d_{IMG} f + f^2 = f^2$$
$$\left(d_{OBJ} - f\right)\left(d_{IMG} - f\right) = f^2 \qquad \text{QED.}$$

6-3. *Calculating a Lens' Focal Length.* A small object 5 cm in front of a lens forms a real image 50 cm behind the lens. What is the lens' focal length? If instead the image were a virtual one 50 cm in front of the lens, then what would be the lens' focal length?

SUGGESTED SOLUTION: Solving the Gauss' formula for focal length:

$$f = \frac{d_{OBJ} d_{IMG}}{d_{OBJ} + d_{IMG}}$$

Putting in the numbers for the real image (image distance positive) ~

$$f = \frac{(5\,\text{cm})(50\,\text{cm})}{5\,\text{cm} + 50\,\text{cm}} = \frac{250\,\text{cm}^2}{55\,\text{cm}} \approx 4.55\,\text{cm} ,$$

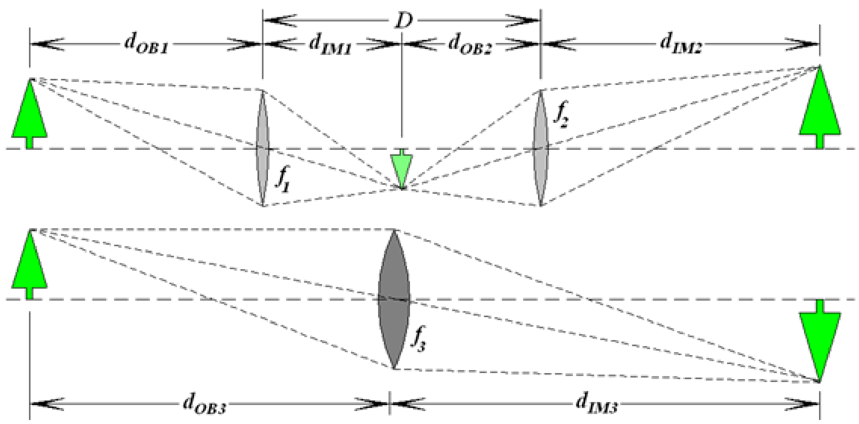
~ and for the virtual image (image distance negative) ~

$$f = \frac{(5\,\text{cm})(-50\,\text{cm})}{5\,\text{cm} - 50\,\text{cm}} = \frac{-250\,\text{cm}^2}{-45\,\text{cm}} \approx 5.56\,\text{cm}$$

Surprise: note that in both cases, the lens is a positive, converging lens. The difference is that the object is outside and inside of the lens' focal length, respectively. (The first case is on the *real image* branch of the plot in Problem 6-2 solutions, and the second case is on the *virtual image* branch.)

6-5. ***Focal Length of Two Lenses.*** Consider two <u>thin</u> lenses, each with focal length f = 100 cm. What is their combined effective focal length if they are (a) touching (i.e., distance between them along the optical axis ≈ 0), or separated by (b) 100 cm, (c) 200 cm, or (d) 400 cm? [Assume light is entering from "infinity."]

SUGGESTED SOLUTION: Use the same general notation scheme as in the last problem (6-4), as shown in the figure ~



(a) For the first part of the problem, let $d_{OB1} = \infty, f_1 = f_2 \equiv f$, and $D = 0$, and apply the Gauss formula:

$$\cancel{\frac{1}{d_{OB1}}} + \frac{1}{d_{IM1}} = \frac{1}{f_1} = \frac{1}{f} \quad \Rightarrow \quad d_{IM1} = f \quad \Rightarrow \quad d_{OB2} = \cancel{D} - d_{IM1} = -f$$

Then do it a second time:

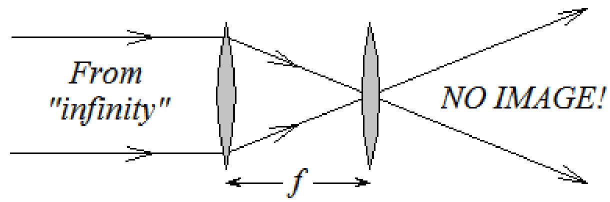
$$\frac{1}{d_{OB2}} + \frac{1}{d_{IM2}} = \frac{1}{f_2} \quad \Rightarrow \quad d_{IM2} = \frac{d_{OB2} f_2}{d_{OB2} - f_2} = \frac{(-f)(f)}{-f-f} = \frac{f}{2}$$

Now assume $d_{OB3} = d_{OB1} = \infty$, $d_{IM3} = d_{IM2} = \frac{f}{2}$, and $f_3 = f_{eff}$ and use the formula for a third time:

$$\cancel{\frac{1}{d_{OB3}}} + \frac{1}{d_{IM3}} = \frac{1}{f_3} = \frac{1}{f_{eff}} \quad \Rightarrow f_{eff} = d_{IM3} = d_{IM2} = \frac{f}{2}$$

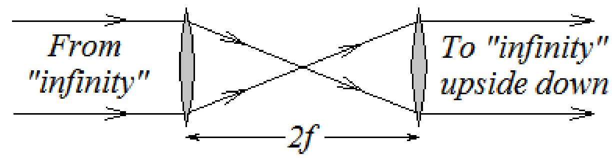
Numerically, then, when the two equal lenses are touching, their combined focal length[1] is $f_{eff} = \frac{100\text{ cm}}{2} = 50\text{ cm}$.

For (b) we still have $d_{IM1} = f$ but since $D = f$ we have $d_{OB2} = f - f = 0$. This makes nonsense out of either Newton's or Gauss' formulas, because $d_{IM2} = 0$ as well. We conclude therefore there is NO SOLUTION for this case (see figure at right).



Part (c) is also a little weird. We still have $d_{OB1} = \infty$ and $d_{IM1} = f$, but this time $d_{OB2} = D - f = 2f - f = f$. This results in

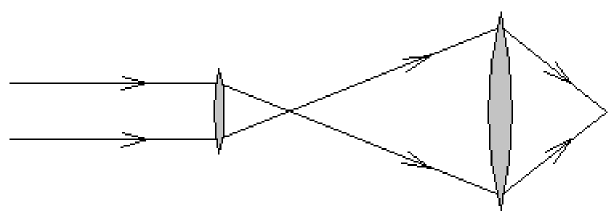
$$d_{IM2} = \frac{d_{OB2} f_2}{d_{OB2} - f_2} = \frac{f^2}{0} = \infty .$$



Thus, Gauss' or Newton's formula tells us that $f_{eff} = \infty$, which is again nonsense. (This would be a lens with no curvature to its surfaces; in other words, just a flat plate of glass.) The picture shows the physical result is that a parallel beam of light is simply inverted. (This is not completely impractical. There may be instances when it is desired to erect an image inverted by other optical elements.)

At last, Part (d) gives a reasonable answer. With $d_{OB2} = D - d_{IM1} = 4f - f = 3f$,

$$d_{IM2} = \frac{d_{OB2} f_2}{d_{OB2} - f_2} = \frac{(3f)(f)}{3f - f} = \frac{3f}{2}$$



Finally, the effective focal length of this lens pair combination is

$$\frac{1}{d_{OB3}} + \frac{1}{d_{IM3}} = \cancel{\frac{1}{\infty}} + \frac{1}{\frac{3f}{2}} = \frac{1}{f_{eff}} \quad \Rightarrow$$

$$f_{eff} = \frac{3f}{2} = \frac{(3)(100\text{ cm})}{2} = 150\text{ cm}.$$

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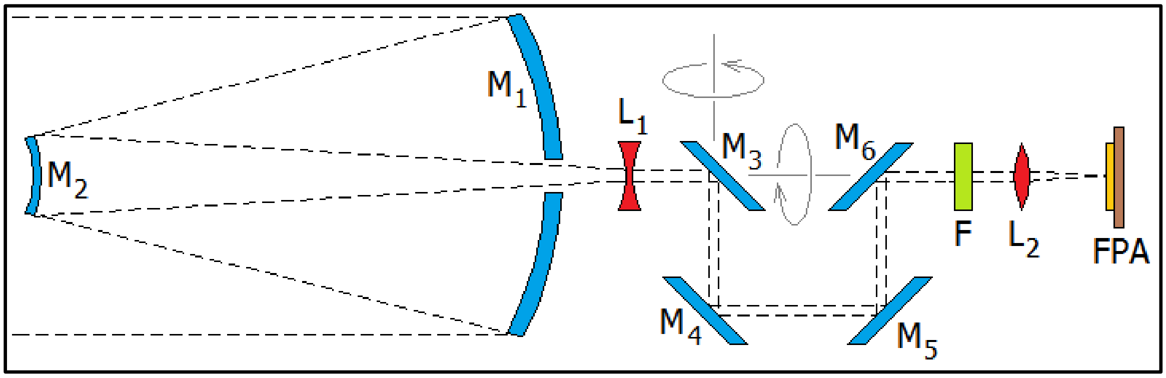
[1] This could have been predicted in the following way. The "power" of a lens is defined as $P = \frac{1}{f\text{ (in meters)}}\ [\text{diopters}]$, and when two lenses are touching their combined power is $P_{TOTAL} = P_1 + P_2$. Thus for two equal lenses, $P_{TOTAL} = \frac{1}{f_{eff}} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{1}{f} + \frac{1}{f} = \frac{2}{f}$, from which $f_{eff} = \frac{1}{P_{TOTAL}} = \frac{f}{2}$. Although standard spectacles do not exactly touch the eye's lens, optometrists use the power of a lens to figure a person's eyeglass prescription. A normal eye should have a power of about $P_{eye} \approx \frac{1}{0.16\,\text{m}} \approx 6.25\,\text{diopters}$, and after measuring a person's eye the optometrist figures the power of the spectacle lens necessary to correct it to normal.



6-7. ***Cassegrain Telescope.*** Consider the arrangement of lenses and mirrors for the remote sensor shown above. The primary collecting mirror M_1 has an aperture of 50 cm and a focal length of 200 cm. Secondary mirror M_2 is located 150 cm in front of M_1, has diameter 12.5 cm, and brings light to a focus 50 cm *behind* M_1 (through the hole). Lens L_1 is 25 cm behind M_1, and changes the converging light from the front mirrors into a collimated, parallel beam. Mirrors M_3, M_4, M_5, and M_6 are flat surfaces which serve as an optical "universal joint," allowing the telescope to swivel or gimbal around two perpendicular axes and bringing the light to a stationary detector. Lens L_2 brings the light to a focus on the electro-optical photodetector, FPA, 25 cm away (from L_2).

A. What is the effective focal length of M_1 and M_2 together, and what is the *f/#*?

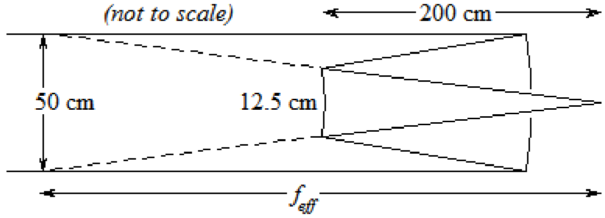
B. Where is the image formed by lens L_1? Since lens L_2 forms the final image on the detector, how far in front of L_2 is the "target" that it is imaging? Combined with M_1 and M_2, what is the overall effective focal length of the telescope system?

C. Filter F limits the bandpass of this sensor to 3.3 – 3.5 μm and has 85% transmission. Mirrors M_1 and M_2 are polished beryllium, M_3, M_4, M_5, and M_6 are dielectric with reflectance 0.985 at 45°; and L_1 and L_2 are made of zinc sulfide with nominal thickness of 1 cm. What is the system's optical transmission factor?

D. Suppose this sensor is used to image the Earth at night from a low altitude orbit of 600 km. Calculate the power in the Airy disk from a 1000 W incandescent light bulb (point source) in the field of view.

SUGGESTED SOLUTION:

A. Drawing the light path for the primary and secondary mirrors (at right, but not to scale), we see there is no need to calculate image or object distances, but we can only use proportional triangles to find the effective focal length ~

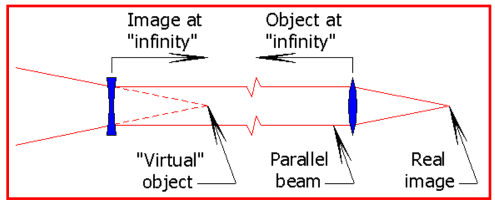


$$\frac{f_{eff}}{50\text{ cm}} = \frac{200\text{ cm}}{12.5\text{ cm}} \quad \Rightarrow \quad f_{eff} = 800\text{ cm}$$

From this, we can find the $f/\#$ to be

$$f/\# = \frac{800\text{ cm}}{50\text{ cm}} = 16$$

B. The problem statement explains that lens L_1 forms a collimated beam, meaning the image it forms is "at infinity." The target lens L_2 is looking at is therefore also "at infinity." Since M_2 forms an image 50 cm behind M_1, but L_1 cuts it off at 25 cm behind M_1, and since the final image formed by L_2 is 25 cm behind L_2, we can reason that the overall effect of L_1, M_3, M_4, M_5, M_6, and L_2 is … nothing. That is, the "universal joint" just takes the last 25 cm of beam path to the final image and moves it away from behind M_1 to a photoelectric detector located somewhere else. The effective focal length of the telescope as a whole is just the 800 cm we found in Part A.



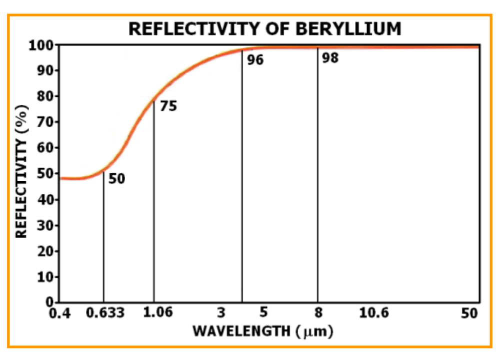
COMMENT: The function of the optical collimator is to transfer energy from one place to another in a <u>parallel</u> beam so as to not lose any energy (by spilling over the edges of lenses or mirrors, but only by minimal reflective losses). Lens L_1 therefore outputs a parallel beam which forms an image at "infinity." Note that the "object" for lens L_1 is the original image formed by the primary and secondary mirror. Since this object is "behind" the lens, the object distance is negative; consequently lens L_1 has a negative focal length and is said to be a negative, or diverging, lens.

$$\frac{1}{d_{OBJ1}} + \frac{1}{d_{IMG1}} = \frac{1}{f_1} \quad \Rightarrow \quad \frac{1}{d_{OBJ1}} + \cancel{\frac{1}{\infty}} = \frac{1}{f_1} \quad \Rightarrow \quad f_1 = d_{OBJ1} = -25\text{ cm}$$

The parallel beam input to lens L_2 is then coming from "infinity." (The two lenses are therefore *complements* of one another, the second reversing action of the first. Since the beam between L_1 and L_2 is a parallel beam, the distance between L_1 and L_2 could be *any* distance, even zero.) We can then find the focal length of the second lens:

$$\frac{1}{d_{OBJ2}} + \frac{1}{d_{IMG2}} = \frac{1}{f_2} \quad \Rightarrow \quad \cancel{\frac{1}{\infty}} + \frac{1}{25\text{cm}} = \frac{1}{f_2} \quad \Rightarrow \quad f_2 = 25\text{cm}$$

Since the original image is formed 25 cm behind L_1, and the final image is formed 25 cm behind L_2, the overall effect is as though the two lenses weren't even there, which is the point of the collimator-universal joint optical assembly.



C. Following photons through the optical system, we first look up the reflectivity of beryllium, as in **Figure 6-19**, replotted at right. Within the bandpass, an average value of reflectivity for polished beryllium appears to be about 96%.

Next, a careful reading of the optical properties of zinc sulfide shown in **Figure 6-12**, shows that its transmission in the bandpass is approximately 0.67. But this is for a nominal thickness of 5 mm, whereas our lenses are 10 mm (1 cm) thick. Assuming a Beer's Law relation for the transmission through any thickness of lens, we can adjust as follows:

$$\tau_{LENS} \approx e^{-\beta\cdot(5\text{mm})} \approx 0.67 \quad \Rightarrow \quad e^{-\beta\cdot(1\text{cm})} = e^{-\beta\cdot(5\text{mm})\times 2} = \left[e^{-\beta\cdot(5\text{mm})}\right]^2 \approx 0.67^2 \approx 0.45$$

Before leaving off our discussion of the lenses, we note that the refractive index[2] of zinc sulfide is approximately $n \approx 2.48$ at 3.34 μm. This would give a surface reflection (**Equation 3-10**) and transmission of

$$\rho_{\perp} = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{1.48}{3.48}\right)^2 \approx 0.181 \quad \text{and} \quad \tau_{\perp} \approx 0.819$$

This is huge! With four surfaces (two lenses) the transmission through the surfaces alone would only be $0.819^4 \approx 0.45$. Because of this, we will assume the lenses have anti-reflection coatings on them, so we can subsequently ignore any small reflection losses.

Taking all the elements in order, the total transmission function is

$$\begin{aligned}\tau_{OPT} &\approx \rho_{M1} \times \rho_{M2} \times \tau_{L1} \times \rho_{M3} \times \rho_{M4} \times \rho_{M5} \times \rho_{M6} \times \tau_{L2} \times \tau_{FILTER} \\ &\approx 0.96 \times 0.96 \times 0.45 \times 0.985 \times 0.985 \times 0.985 \times 0.985 \times 0.45 \times 0.85 \approx 0.15\end{aligned}$$

COMMENT: Evidently there must be some very good reason for using the zinc sulfide lenses, because otherwise their optical transmission kills the signal.

D. Begin this part of the problem by assuming the Φ_{BULB} = 1000 W light bulb is an incandescent lamp with a tungsten filament at a temperature of $T \approx 2400$ K. Assuming it is a graybody with constant emissivity, the fraction of its power emitted in our bandpass (limited by the filter to 3.3 – 3.5 μm) is ~

$$\Phi_{IN-BAND} \approx \left(\frac{\int\limits_{BANDPASS} B_{\lambda}(T)\, d\lambda}{\sigma_{SB} T^4}\right) \Phi_{BULB}$$

Its intensity is then $I_{IN-BAND} \approx \dfrac{\Phi_{IN-BAND}}{4\pi}$, which gives us an irradiance at aperture of $\mathrm{E} \approx \dfrac{I_{IN-BAND}}{R^2}\tau_{ATM}$. The power through the aperture is then $\Phi_{APERTURE} \approx \mathrm{E}\, A_{APERTURE}$ $\approx \mathrm{E}\left(\dfrac{\pi D_1^2}{4} - \dfrac{\pi D_2^2}{4}\right)$ where we take into account the obscuration caused by the secondary mirror.

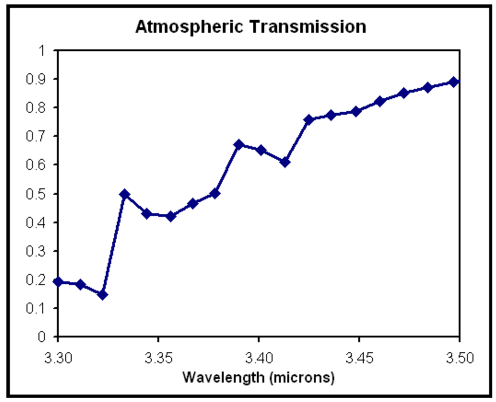
Finally, the power reaching the focal plane is $\Phi_{FPA} \approx \Phi_{APERTURE}\tau_{OPT}$ and the power in the Airy disk is $\Phi_{PSF} \approx 0.84\Phi_{FPA}$ image is. Putting it all together ~

$$\begin{aligned}\Phi_{PSF} &= 0.84\Phi_{FPA} = 0.84\Phi_{APERTURE}\tau_{OPT} = 0.84\mathrm{E}\, A_{APERTURE}\tau_{OPT} = \mathrm{E}\left[\frac{\pi D_1^2}{4} - \frac{\pi D_2^2}{4}\right]\tau_{OPT} \\ &= 0.84\frac{I_{IN-BAND}}{R^2}\tau_{ATM}\left[\frac{\pi D_1^2}{4} - \frac{\pi D_2^2}{4}\right]\tau_{OPT} = 0.84\frac{\Phi_{IN-BAND}}{4\pi R^2}\tau_{ATM}\left[\frac{\pi D_1^2}{4} - \frac{\pi D_2^2}{4}\right]\tau_{OPT}\end{aligned}$$

2 http://refractiveindex.info/?group=CRYSTALS&material=ZnS.

$$= 0.84 \frac{\int B_\lambda(T)\,d\lambda}{\sigma_{SB} T^4} \frac{\Phi_{BULB}}{4\pi R^2} \tau_{ATM} \left[\frac{\pi D_1^2}{4} - \frac{\pi D_2^2}{4} \right] \tau_{OPT} = \frac{0.84\left(\int B_\lambda(T)\,d\lambda\right)\Phi_{BULB}\tau_{ATM}\left(D_1^2 - D_2^2\right)\tau_{OPT}}{16\sigma_{SB} T^4 R^2}$$

What is slightly askew in this last expression is an improper treatment of wavelength dependence. Technically, the atmospheric and optical transmission functions are functions of wavelength, so should be included in the integral. We have already calculated τ_{OPT}, which we tacitly assumed to be constant over the bandpass. Not so τ_{ATM}; the plot at right shows it to have considerable variation. Therefore, a more correct calculation is



$$\Phi_{PSF} \approx \frac{0.84\left(\int B_\lambda(T)\tau_{ATM}\,d\lambda\right)\Phi_{BULB}\left(D_1^2 - D_2^2\right)\tau_{OPT}}{16\sigma_{SB} T^4 R^2}$$

To complete the calculation, the integral is computed in the companion spreadsheet, and all the other values (known or assumed) can be plugged in to give ~

$$\Phi_{PSF} \approx \frac{(0.84)\left(2.02\times10^4\,\text{W/m}^2\right)(10^3\,\text{W})\left[(0.50\,\text{m})^2 - (0.125\,\text{m})^2\right](0.15)}{(16)\left(5.67\times10^{-8}\,\text{W/(m}^2\text{K}^4)\right)(2400\,\text{K})^4(6\times10^5\,\text{m})^2} \approx 5.5\times10^{-14}\,\text{W}$$

MORE CONSIDERATIONS: This doesn't seem like a lot of power, but let's calculate how many photons this is. The energy per photon is approximately

$$E_{PHOTON} = \frac{hc}{\lambda} \approx 5.8\times10^{-20}\,\text{J},$$

using an average wavelength of 3.4 μm. This gives the rate of photons being received as

$$\frac{5.5\times10^{-14}\,\text{W}}{5.8\times10^{-20}\,\text{J}} \approx 9.5\times10^5\,\text{photons/s},$$

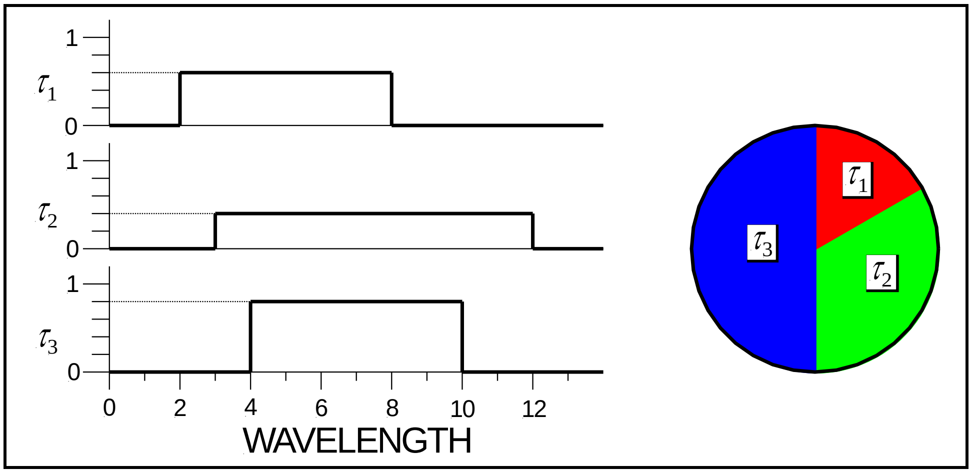
which is more than an adequate number to detect. (However, we have ignored the part of the question where it says that this collection takes place at night – we really should calculate the background radiation to see if we can detect the light bulb against it. The worst case background in this bandpass would most likely be reflected moonshine.)

One more question we should probably ask is "What's the size of the PSF?" A straightforward calculation is

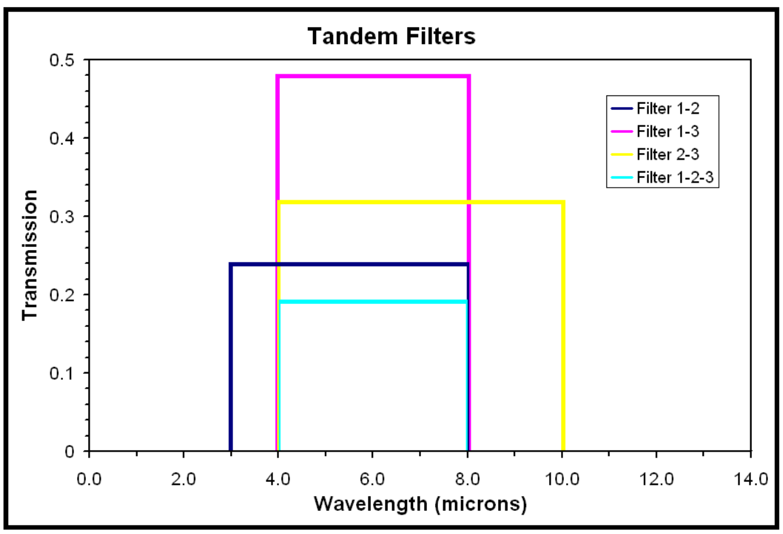
$$2r_A \approx 2\frac{1.22 f\lambda}{D} = \frac{(2)(1.22)(8.00\,\text{m})(3.4\,\mu\text{m})}{0.50\,\text{m}} \approx 133\,\mu\text{m diameter}$$

Comparing this to the size of a nominal pixel – about 10 μm – we estimate the PSF is therefore more than 13 pixels wide! If we estimate the number of pixels within a PSF as roughly $\pi\times(6.5\,\text{pixels})^2 \approx 130\,\text{pixels}$, then the rate of photons falling on a pixel within the PSF is, on the average, about 7300 photons per second. There will be more photons falling on central pixels, and less on the outer pixels, of course, but this should still be sufficient for detection.

6-9A. ***Combinations of Filters.*** Consider three separate filters with the transmission functions plotted at left in the figure below. If two of these filters were selected for a remote sensor and were placed in the optical collection system in tandem (one after the other), calculate and plot the transmission if filters 1 and 2 were selected. Repeat for filters 1 and 3. Repeat again for filters 2 and 3. Does it matter in which order the filters are placed into the optical system? Finally, calculate and plot the transmission function if all three filters were used.

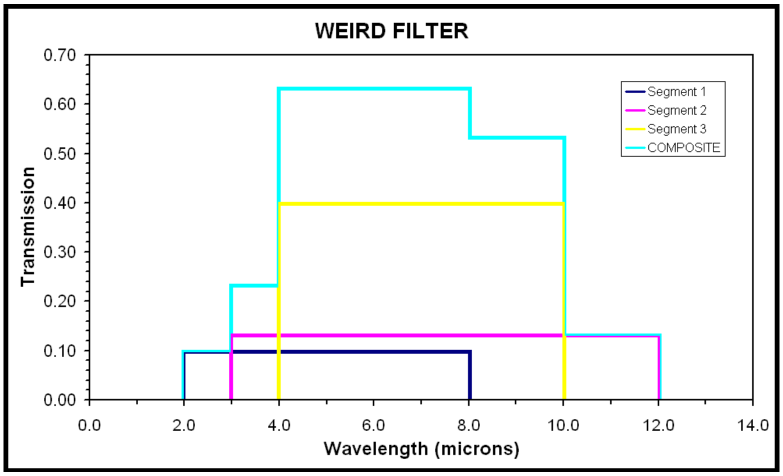


SUGGESTED SOLUTION: The light propagated through filters in tandem is the ***product*** of the transmissions through the individual filters. The order of the filters does not matter. See the spreadsheet where we have calculated the transmission through tandem filters in pairs and for all three. Here's the plot:



6-9B. ***A Weird Composite Filter.*** Again refer to the filter transmission functions at left in the figure above, *and* the sketch on the right of a weird composite filter made from sections of the three individual filters: a 60º sector from filter 1, a 120º sector from filter 2, and a 180º sector from filter 3. Calculate and plot the transmission function for this composite filter.

SUGGESTED SOLUTION: Things are a little different this time. One-sixth of all the photons incident on the composite filter will pass through the first segment, one-third through the second segment, and one-half through the third segment. Of those photons, only those with wavelengths within the bandpasses of the segments are transmitted, and then only in the amount of the transmission of their respective segments. In all, the number of photons making it through at any wavelength is the ***sum*** of those that have passed through the three segments. The calculations are in the companion spreadsheet, and here's the plot.



6-11. ***A BIG Camera.*** The camera pictured here is on display at the National Museum of the United States Air Force, Wright-Patterson Air Force Base, Riverside, Ohio. The placard on the display (in lower left of photograph) reads:

The "Boston Camera"

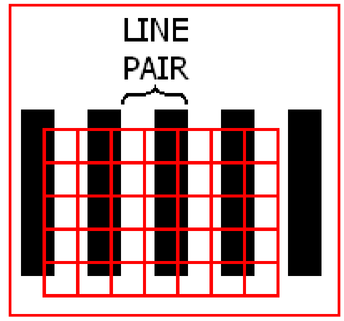
This camera, manufactured for the US Air Force by Boston University in 1951, is the largest aerial camera ever built. It was installed in an RB-36D in 1954 and tested for about a year. Later it was used in a C-97 aircraft flying along the air corridor through communist East Germany to Berlin, but a 10,000 ft altitude restriction imposed by the communists made the camera less useful than at a higher altitude. It was also used on reconnaissance missions along the borders of Eastern European nations. The camera made an 18 × 36 inch negative and was so powerful that a photo interpreter could detect a golf ball from an altitude of 45,000 feet. Dr. James Baker of Harvard University designed the camera.

TECHNICAL NOTES

Shutter: focal plane, fixed slit, pneumatic drive, electrically tripped
Shutter speed: 1/400 sec
Resolution: 28 L/mm (lines per millimeter)
Weight: 6500 lbs (camera and aircraft mount)

Note that the inset in the photograph shows the focal length and f-stop marked on the camera's primary lens. Using these data and the flight information suggested by the placard, calculate the size of a golf ball's image on the camera's film plane. Calculate the size of the PSF and compare to the image's size.

SUGGESTED ANSWER: The writing on the front of the camera indicates that its focal length is 240″ (= 20′) and its f-number is *f*/8. (This would give an aperture diameter of $\frac{240''}{8} = 30''\ (= 2.5')$, which looks about right from the picture.) Assuming the camera is pointing straight down, we'll take the dimensions of its focal plane (film) to be $x = 18'' = 1.5'$ and $y = 36'' = 3'$ as stated. Its IFOV is then ~



$$\text{From } 10{,}000' \left\{ \begin{aligned} X &= \frac{Hx}{f} = \frac{(10{,}000')(1.5')}{20'} \approx 750' \\ Y &= \frac{Hy}{f} = \frac{(10{,}000')(3')}{20'} \approx 1500' \end{aligned} \right\} \approx 1.13 \times 10^6 \,\text{sq ft}$$

$$\text{From } 45{,}000' \left\{ \begin{aligned} X &= \frac{Hx}{f} = \frac{(45{,}000')(1.5')}{20'} \approx 3375' \\ Y &= \frac{Hy}{f} = \frac{(45{,}000')(3')}{20'} \approx 6750' \end{aligned} \right\} \approx 2.28 \times 10^7 \,\text{sq ft}$$

Since we are MASINTers, we probably don't appreciate the IMINTers' need to see larger areas to literally identify more activities and objects. Although the IFOV from 10,000′ seems impressive to us, it is probably insufficient for monitoring borders.

The question of GSD is not quite as simple. Since it is claimed a PI (now called "IA: Image Analyst") can resolve 28 lines per millimeter, what is meant is that you can resolve 28 *line pairs* per millimeter where a "line pair" is a white and a black line. Nyquist's criterion specifies that we must sample the line pair with TWO pixels. (In this case, a pixel is a number of film grains, but we don't want to go there.) That means we need to have 56 pixels per mm or 17.9 µm per pixel on our focal plane. This gives us ~

$$\text{For } 10{,}000' \sim \text{GSD} = \frac{10{,}000'}{20'} 17.9\,\mu\text{m} \approx 8.95\,\text{mm}$$
$$\text{For } 45{,}000' \sim \text{GSD} = \frac{45{,}000'}{20'} 17.9\,\mu\text{m} \approx 40.3\,\text{mm}$$

This last result more or less agrees with the placard's statement that we could detect a golf ball[3] from 45,000′, although there is some room for interpretation what is meant by the word "detect." Common usage in the IMINT world of such words (including "resolution") differs somewhat from what we have learned in the non-literal world of MASINT.

SOME COMMENTS: There are at least two more issues that we might want to address before leaving this camera behind. First, motion compensation: the aircraft platform, a C-97, is probably flying at about 210 kt. (This is estimated from the author's experience of having refueled behind a KC-97 tanker a couple of times.) Its ground speed – and the motion of the IFOV over the ground – is about ~

[3] United States Golf Association (USGA) rules specify that a golf ball shall be no smaller than 42.67 mm diameter.

$$\frac{210\,\mathrm{NM}}{\mathrm{hr}}\times\frac{10,000\,\mathrm{km}}{5400\,\mathrm{NM}}\times\frac{1\mathrm{hr}}{3600\,\mathrm{s}}\approx 108\,\mathrm{m/s}$$

For the given shutter speed (1/400 s), the camera moves forward a distance of ~

$$108\,\mathrm{m/s}\times 1/400\,\mathrm{s}\approx 27\,\mathrm{cm}$$

Obviously, this is much greater than the alleged GSD, so the camera needs to have some form of motion compensation to hold the image "still." This can be accomplished by either rotating the camera on a pivot (which appears likely judging from the huge spindle sticking out the side of the camera) or the focal plane (film) could be moved in a proportional amount opposite to the motion.

Second, supposing the camera is being flown along the border at 10,000′ and 210 kt. For maximum coverage, we want to orient the FOV such that we are taking pictures 1500′ wide (cross-track) by 750′ (≈ 230 m) high (along track). This gives us only a little over two seconds between exposures to change the film. Perhaps this is another reason why flying at the lower altitude is not as effective.