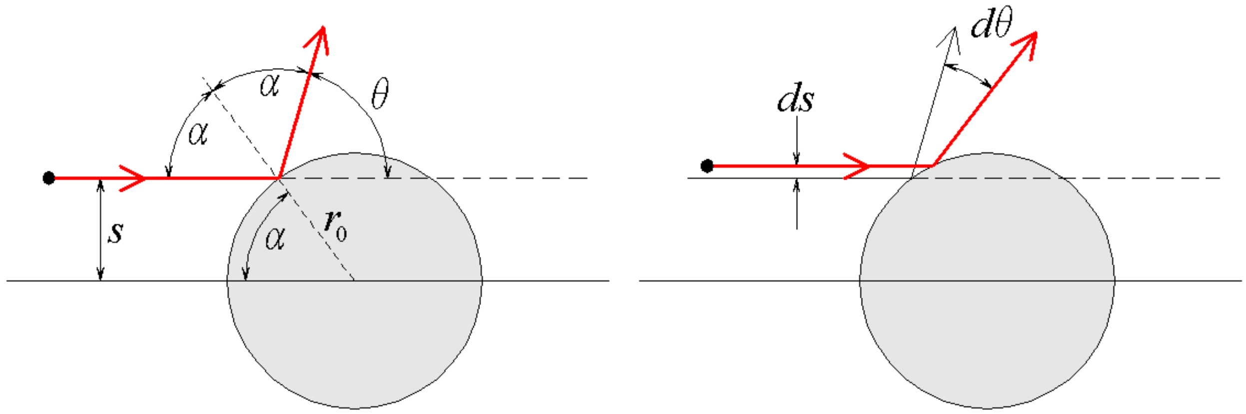
APPENDIX 5-6: Hard Sphere Scattering



For incident "bullet" particles of negligible size, the apparent cross-section as a function of solid angle *scattered into* is

$$\frac{d\sigma}{d\Omega} \approx \frac{s\,ds}{\sin\theta\,d\theta} = \frac{-r_0^2}{4} \tag{1}$$

(The significance of the minus sign is that the scattering angle, θ, decreases as *impact parameter, s*, increases. This will be ignored in the following calculations.)

For all intents and purposes, every bullet that hits the sphere is scattered out of the line of sight, so the total scattering cross-section is

$$\sigma_{SCAT} \approx \int_{ALL\,\Omega} \frac{d\sigma}{d\Omega}\,d\Omega \approx \int_0^{2\pi}\int_0^{\pi} \frac{r_0^2}{4}\sin\theta\,d\theta\,d\varphi \approx \pi r_0^2 \tag{2}$$

This is just the answer the reader would expect – the physical cross-section of the target sphere. The reader is invited to discover the answer for bullets of finite radius, r_B, impinging on a target sphere of radius r_T. [HINT: a simple substitution will do.]

In another discipline, monostatic radar, the interest is not in how many bullets are scattered out of a path, but only the number of bullets directly *back-scattered* to the receiver. This is easily calculated:

$$\sigma_{BS} \equiv \beta \approx \int_{\theta=\pi} \frac{d\sigma}{d\Omega}\,d\Omega \approx \int \frac{d\sigma}{d\Omega}\,\delta(\theta-\pi)\,d\Omega \approx \left.\frac{d\sigma}{d\Omega}\right|_{\theta=\pi} \approx \frac{r_0^2}{4} \approx \frac{\sigma_{SCAT}}{4\pi} \tag{3}$$