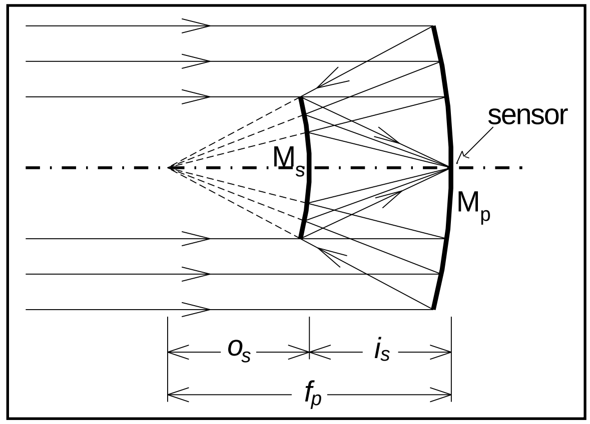
SUGGESTED SOLUTIONS (ODD)

CHAPTER 8

NOTE: Use three-digit precision for all calculations unless otherwise stated or implied.

8-1. *Calculating Field of View.* Recall the two-mirror telescope system we analyzed in Problem 6-6 (shown at right: primary aperture $D = 30.0\ \text{cm}$, and effective focal length $f_{eff} \approx 2.16\ \text{m}$). If the photo-detector at the telescope's focus is a 1024 × 1024 pixel focal plane array with 10 μm pitch, what is its FOV? If the sensor was nadir pointing, what would its altitude have to be so that its GSD is not less than one meter? What would the sensor's altitude have to be such that it can spatially resolve two point sources on the ground separated by one meter?



SUGGESTED SOLUTION: The dimensions of the FPA are

$$x_{FPA} = y_{FPA} = 1024\ \text{pixels} \times 10\ \mu\text{m/pixel} \approx 10{,}240\ \mu\text{m} = 1.024\times10^{-2}\,\text{m},$$

so the FOV is

$$\Omega_{FOV} = \frac{xy}{f_{eff}^2} = \frac{\left(1.024\times10^{-2}\,\text{m}\right)^2}{\left(2.16\ \text{m}\right)^2} \approx 2.25\times10^{-5}\text{sr}.$$

By proportional triangles we can find the altitude for a one meter GSD, noting that the dimension of one pixel is approximately equal to the pitch (10 μm = 10^{-5} m):

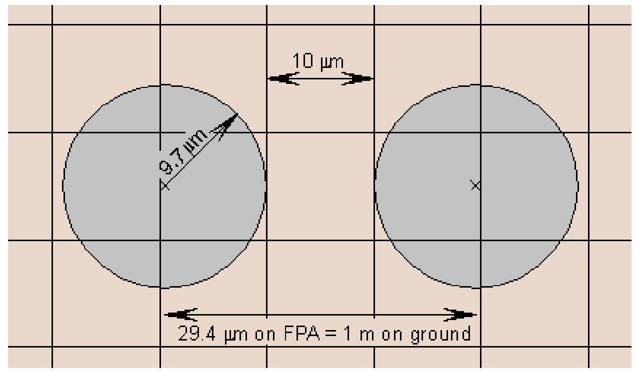
$$\frac{GSD}{h} = \frac{x_{PIX}}{f_{eff}} \quad \Rightarrow \quad h = \frac{GSD\cdot f_{eff}}{x_{PIX}} = \frac{(1\ \text{m})(2.16\ \text{m})}{10^{-5}\text{m}} \approx 2.16\times10^{5}\text{m, or } 216\ \text{km}.$$

(This would put the sensor on a fairly low altitude spacecraft, whose orbit would probably decay within a year. See Figure 4-33.)

The apparent answer to the last question is to invoke the Nyquist criterion, requiring that two GSDs must fit between the two point sources. That would put the sensor at one-half the altitude calculated above, or 108 km, which would certainly decay in less than a month on orbit. <u>However</u>, there is another consideration before pronouncing a final answer: consider the PSF.

Since this sensor seems to be at fairly low altitude, we can assume that its primary mission is to collect imagery in the visible to near infrared (VNIR). This would suggest that it probably has a silicon photodetector, which is sensitive to wavelengths as long as 1.1 μm. (The mirrors are most likely polished aluminum, and the atmospheric transmission is not a severe limitation.) The PSF radius is therefore approximately

$$r_A \approx \frac{1.22 f_{eff} \lambda}{D} = \frac{(1.22)(2.16\,\mathrm{m})(1.1\,\mu\mathrm{m})}{0.30\,\mathrm{m}} \approx 9.7\,\mu\mathrm{m}$$



which is almost the same size as a pixel! We must therefore abandon the notion of being able to resolve two point sources by the Rayleigh criterion because we cannot simultaneously satisfy the Nyquist criterion. We must fall back on the IMINTer's spatial resolution criterion requiring one pixel (one GSD on the ground) between the images of the two sources to contain none of their energy. This leads us to the situation shown in the sketch ~ the centers of the two PSFs must be separated by at least $x_{SEPARATION} \approx 29.4\,\mu\mathrm{m}$ on the FPA. Using this as our resolution criterion, the sensor's altitude can then be figured as before to be:

$$h = \frac{\{\text{separation on ground}\}\, f_{eff}}{\{\text{separation on FPA}\}} = \frac{(1\,\mathrm{m})(2.16\,\mathrm{m})}{2.94\times10^{-5}\,\mathrm{m}} \approx 7.35\times10^{4}\,\mathrm{m},\ \text{or} 73.5\ \mathrm{km}\ .$$

This is below the usable spacecraft altitude, but above the service ceiling for aircraft. Note that the choice of the longest wavelength is the worst case for resolution, and is the highest altitude to assure image separation. Any shorter wavelength could be resolved at a lower altitude.

8-3. *Diffraction Limited Optics*. A certain sensor is designed to resolve between two point targets 1.0 m apart at a distance of 100 km in 2.0 µm IR light. What is the size of its diffraction-limited aperture?

SUGGESTED SOLUTION:

$$\theta_{MIN} \approx \frac{1.22\lambda}{D} = \frac{X}{R} \quad \Rightarrow \quad D = \frac{1.22\lambda R}{X} = \frac{(1.22)(2.0\times10^{-6}\,\mathrm{m})(10^{5}\,\mathrm{m})}{1.0\,\mathrm{m}} \approx 24.4\,\mathrm{cm}$$

There is not enough information given in the problem to tell if the Nyquist criterion has also been satisfied.

8-5. ***Theoretical Resolution and Footprint.*** A remote sensor consists of a single spherical mirror with a focal plane array located at its focus (this is called a Newtonian telescope). The focal length of the mirror is 60 cm and its diameter is 30 cm. The one megapixel FPA is 1 cm^2.

A. What is the telescope's theoretical angular resolution for 2.5 μm SWIR light?

B. Can this sensor actually resolve two targets separated by the minimum angular resolution?

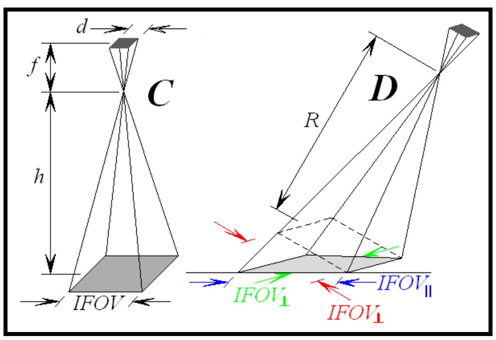
C. Looking directly downward from an altitude of 600 km, what is the IFOV (focal plane footprint) and GSD (pixel footprint)?

D. Looking at a nadir angles of 15°, 30°, 45°, 60°, and 75° (from 600 km altitude), what is the GSD of a pixel on boresight?

SUGGESTED SOLUTION:

A. Assuming Rayleigh criterion: $\theta_{MIN} = \dfrac{1.22\lambda}{D} = \dfrac{(1.22)(2.5\times10^{-6}\,\text{m})}{0.30\,\text{m}} \approx 1.02\times10^{-5}\,\text{rad}$.

B. The Airy disk (blur circle) size is $r_A = f\,\theta_{MIN} = (0.6\,\text{m})(1.02\times10^{-5}) \approx 6.1\,\mu\text{m}$, while the (assumed square) pixel dimension is $d = \dfrac{0.01\,\text{m}}{\sqrt{10^6}} \approx 10\,\mu\text{m}$. So obviously this sensor will not be able to perform at its theoretical maximum resolution.



C. Referring to the left sketch in the figure at left, similar triangles give us $\dfrac{IFOV}{d} = \dfrac{h}{f}$, so ~

$$IFOV = \frac{h\,d}{f} = \frac{(600\,\text{km})(0.01\,\text{m})}{0.60\,\text{m}} \approx 10\,\text{km}.$$

GSD (one pixel's worth of IFOV) is therefore ~ $GSD = \dfrac{10\,\text{km}}{\sqrt{10^6}} \approx 10\,\text{m}$.

D. [Now see the right sketch in the figure above; since it was resurrected from an older version of this problem, the labeling "IFOV" needs to be changed to "GSD" to apply to the present case.] When viewing the ground at a nadir angle, θ_N, the GSD perpendicular (cross-track) to the line of sight is still given by similar triangles (see Equations 8-8 in text):

$$\frac{GSD_\perp}{x_{PIXEL}} = \frac{R}{f} = \frac{h}{f\cos\theta_N} \quad\Rightarrow\quad GSD_\perp = \frac{h\,x_{PIXEL}}{f\cos\theta_N},$$

and the GSD parallel to the line of sight (along-track) is given by

$$GSD_\parallel = \frac{GSD_\perp}{\cos\theta_N} = \frac{h\,x_{PIXEL}}{f\cos^2\theta_N}$$

The following table gives the values for $GSD_{\perp}$ and $GSD_{\parallel}$, as well as the approximate area (Equation 8-9 in the text), for increasing nadir angle. Note there becomes considerable distortion at larger angles.

Nadir Angle	$GSD_{\perp}$	$GSD_{\parallel}$	AREA
(deg)	(m)	(m)	(m^2)
0	10.0	10.0	100
15	10.4	10.7	111
30	11.5	13.3	154
45	14.1	20.0	283
60	20.0	40.0	800
75	38.6	149	5760

8-7. ***An Airborne Sensor.*** A small remotely piloted vehicle (RPV) has a primitive surveillance system consisting of a single lens and a FPA with an infrared filter (0.7 – 1.1 μm). The camera is built into the fuselage with a flat, two-axis gimbaled mirror in the nose. The distance from lens to FPA is 64 cm, the lens is 2 inches in diameter, and the FOV is 1.53×10^{-3} sr (defined by the square shape of the FPA). When flying at 2,000 feet above ground level (AGL), what is the IFOV for nadir viewing and for viewing 30º ahead of the aircraft? Theoretically, how many objects, aligned in a row perpendicular to the ground track, could be resolved across the sensor's FOV at nadir and 30º viewing? For this resolution, what is the required pixel pitch, and how large is the FPA (physical size and number of pixels)?

SUGGESTED SOLUTION: When the sensor looks at nadir, the area it sees – its IFOV – is ~

$$A_{NADIR} \approx R^2\Omega = (609.6\text{ m})^2(1.53\times10^{-3}\text{sr}) \approx 569\text{ m}^2 = \left(IFOV\right)^2 \approx (23.8\text{m})^2$$

When viewing at 30° off nadir, the area it sees is approximately ~

$$A_{30^\circ} \approx \frac{A_{NADIR}}{\cos^3\theta_N} = \frac{569\text{ m}^2}{\cos^3 30^\circ} \approx 875\text{ m}^2 = \left(IFOV_{\perp}\right)\times\left(IFOV_{\parallel}\right) \approx 27.5\text{ m}\times 31.8\text{ m}$$

where we have used the formulae of Equation 8-8 to calculate the IFOV dimensions. (Also see the sketch in Problem Solution 8-5.)

For Rayleigh criterion spatial resolution, the minimum resolvable angle is

$$\theta_{MIN} = \frac{1.22\lambda}{D} = \frac{(1.22)\left(1.1\times10^{-6}\text{m}\right)}{5.08\times10^{-2}\text{m}} \approx 2.64\times10^{-5}\text{rad}$$

where we use the longest wavelength in the bandpass. (Targets at shorter wavelengths could be resolved at smaller angles, of course, but then any longer wavelength targets would not be resolvable at the smaller angles.) Taking the sensor's FOV to have a square cross-section, its angular dimension is

$$\theta_x = \theta_y \approx \sqrt{1.53\times10^{-3}\text{sr}} \approx 2.91\times10^{-2}\text{ rad}$$

The number of resolvable targets across the sensor's FOV is therefore

$$\frac{3.91\times10^{-2}\,\text{rad}}{2.64\times10^{-5}\,\text{rad}} \approx 1480\ \text{targets}$$

This is independent of the targets' distances from the sensor and the direction the sensor is pointing.

For the last part of this problem, we can answer how big is the FPA by again looking at the FOV ~

$$\Omega \approx \frac{A_{FPA}}{f^2} = \frac{x_{FPA}^2}{f^2} \quad \Rightarrow \quad x_{FPA} \approx f\sqrt{\Omega} = \left(0.64\ \text{m}\right)\sqrt{1.53\times10^{-3}\text{sr}} \approx 2.50\ \text{cm}$$

Since we know we want to be able to resolve 1480 objects, the Nyquist criterion says we need twice that number of pixels (2960 pixels). The pitch is therefore ~

$$x_{PIXEL} = \frac{x_{FPA}}{N} = \frac{2.50\ \text{cm}}{2960\ \text{pixels}} \approx 8.44\ \mu\text{m}$$

This is fully within the current state of the art.