SUGGESTED SOLUTIONS (ODD)

CHAPTER 17

NOTE: Use three-digit precision for all calculations unless otherwise stated or implied.

17-1. A glass prism, with an apex angle of 60 degrees, has an index of refraction for blue light of 1.55 and for red light of 1.45. What is the difference in minimum angular dispersion between blue and red wavelengths when the prism is used in air?

SUGGESTED SOLUTION:

Using prism equation

$$\frac{n_\rho}{n_A} = \frac{\sin\left(\frac{\alpha+\delta}{2}\right)}{\sin\left(\frac{\alpha}{2}\right)}$$

For blue

$$\sin\left(\frac{\alpha+\delta_B}{2}\right) = \frac{n_\rho}{n_A}\sin\frac{\alpha}{2}$$

$$= \frac{1.55}{1.00}\sin(30°)$$

$$= 0.775$$

$$\frac{60+\delta_B}{2} = 50.81°$$

$$\delta_B = 41.62°$$

For red

$$\sin\left(\frac{\alpha+\delta_R}{2}\right) = \frac{n_\rho}{n_A}\sin\left(\frac{\alpha}{2}\right)$$

$$= \frac{1.45}{1.00}\sin(30°)$$

$$= 0.725$$

$$\frac{60+\delta_R}{2} = 46.47°$$

$$\delta_R = 32.93°$$

Difference $\delta_B - \delta_R = 8.69°$

Alternate approach

$$\frac{d\delta}{dn_\rho} = \frac{2 \sin\left(\frac{\alpha}{2}\right)}{\sqrt{n_a^2 - n_\rho^2 \sin^2\left(\frac{\alpha}{2}\right)}}$$

$$\Delta\delta = \frac{2 \sin\left(\frac{\alpha}{2}\right) \Delta n_\rho}{\sqrt{n_a^2 - n_\rho^2 \sin^2\left(\frac{\alpha}{2}\right)}}$$

$$= \frac{2 \sin (30)(1.55 - 1.45)}{\sqrt{1 - \left(\frac{1.55 + 1.45}{2}\right)^2 \sin^2 30}}$$

$$= \frac{2\,(0.5)(0.1)}{\sqrt{1 - (1.5)^2\ (0.5)^2}}$$

$$= \frac{0.1}{\sqrt{1 - 0.5625}}$$

$$= \frac{0.1}{\sqrt{0.4375}}$$

$$= \frac{0.1}{0.6614}$$

$$= 0.151 \text{ rad or } 8.65°$$

17-3. A spectrum of white light is obtained with a grating ruled with 2500 lines / cm. What is the angular separation between green (520nm) and red (630 nm) in the second order?

SUGGESTED SOLUTION:

Direct Approach

$$\sin\theta = \frac{m\lambda}{d} \qquad d = \frac{1 \text{ cm}}{2500} = \frac{10^{-2}\text{m}}{0.25 \text{ x } 10^4} = 4 \text{ x } 10^{-6}\text{m}$$

For green

$$\sin\theta_G = \frac{2\,[520\text{nm}]}{4 \text{ x } 10^{-6}\text{m}}$$

$$\theta_G \approx \sin\theta_G = 0.260 \text{ rad}$$

$$\theta_G = 15.1°$$

For red

$$\theta_R \approx \sin\theta_R = 0.315 \text{ rad}$$
$$\theta_R = 18.4°$$

$$\theta_R - \theta_G = 3.3°$$

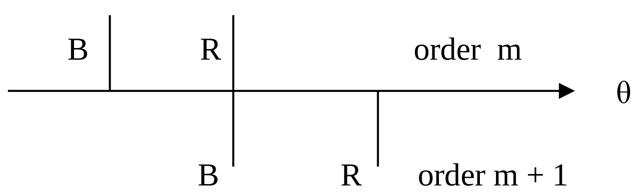
Alternate Approach

$$\Delta\theta \approx \Delta\lambda \left[\frac{m}{d}\right]$$
$$= \frac{[630\text{nm} - 520\text{nm}]2}{4 \times 10^{-6}\text{m}}$$

$$= 0.055 \text{ rad or } 3.15°$$

17-5. Red light at wavelength 600 nm, when diffracted by a grating in a given order, just coincides in angular location with blue light at wavelength 450 nm, diffracted by the same grating in the next higher order. What are the two orders?

SUGGESTED SOLUTION:



$$\sin\theta = \frac{m\lambda}{d}$$

At θ of overlap

$$\sin\theta = \frac{m\,\lambda_R}{d} = \frac{(m+1)\lambda_B}{d}$$

Hence,

$$m\lambda_R = (m+1)\,\lambda_B$$

$$m = \frac{\lambda_B}{\lambda_R - \lambda_B}$$

$$= \frac{450}{600 - 450}$$

$$m = 3$$

$m + 1 = 4$ (results must be integers)

17-7. The two surfaces of a Fabry Perot etalon having an air gap are kept 10 mm apart by a barium titanate spacer around the edges. The Fabry-Perot interferometer is designed to operate at normal incidence at 0.546 micrometers when no voltage is applied.

A. When a voltage of 1.6 kV is applied across this spacer, its length increases to 11 mm. If this is the maximum length change possible for the spacer, what is the spectral band pass of the interferometer?

B. What is the free-spectral-range, assuming $\lambda_s = 0.546\ \mu m$ and using the value of m calculated from the given data above? Can the value for the spectral band pass that was calculated in part a really be this large?

C. If the surface reflectivity at the air etalon surface is 0.8, how closely can two lines be separated and still be resolved?

SUGGESTED SOLUTION:

Fabry Perot

A) $2nd \cos\beta = m\lambda$

For normal incidence, $\beta = 0$ $\quad \cos\beta = 1.0$

For $n = 1.0$, $\qquad d = \frac{m}{2}\lambda$

$$d_1 = \frac{m}{2}\lambda_1 \qquad d_1 = 10\text{mm} \quad \lambda_1 = 0.546\mu\text{m}$$

$$d_2 = \frac{m}{2}\lambda_2 \qquad d_2 = 11\text{mm} \quad \lambda_2 = ?$$

Either solve the first equation for "m", plug it into the second equation and solve for λ_2, OR take ratios (divide the second equation by the first equation).

$$\frac{d_2}{d_1} = \frac{\lambda_2}{\lambda_1}$$

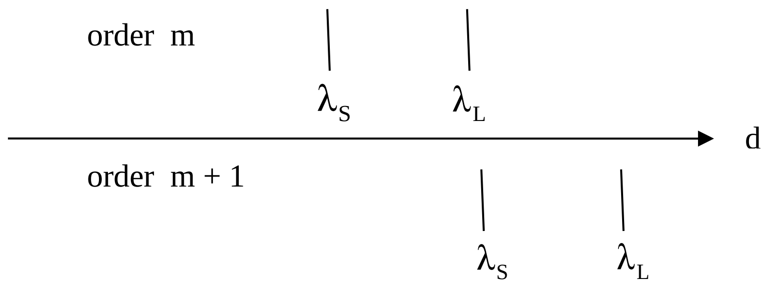
$$\lambda_2 = \lambda_1\left(\frac{d_2}{d_1}\right)$$

$$\lambda_2 = 0.546\left(\frac{11}{10}\right)$$

$$= 0.601\mu\text{m} \quad \Rightarrow \quad \lambda_2 - \lambda_1 = 0.0546\mu\text{m}$$

$$= 54.6\text{nm}$$

B) For free spectral range,



Since, $d = \frac{m}{2}\lambda$, for no overlap

$$d = \frac{m}{2}\lambda_L = \frac{(m+1)}{2}\lambda_S$$

$$m\lambda_L = (m+1)\lambda_S$$

$$FSR = \lambda_L - \lambda_S = \frac{\lambda_S}{m}$$

$$d_1 = \frac{m}{2}\lambda_1 \qquad d_1 = 10mm \qquad \lambda_1 = 546nm$$

$$m = \frac{2d_1}{\lambda_1}$$

$$= 2\,\frac{\left(10\,\cancel{mm}\ \text{x}\ 10^3\mu m/\ \cancel{mm}\right)}{0.546\mu m}$$

$$= 36{,}630$$

$$FSR = \frac{546\ nm}{36{,}630}$$

$$= 0.0149\ nm \quad << \quad \lambda_2 - \lambda_1 = 54.6\ nm \text{ (from part a)}$$

C) Spectral resolution

$$\Delta\lambda_{min} = \frac{\lambda\,(1-\rho)}{m\,\pi\,\sqrt{\rho}}$$

$$= \frac{(546nm)(1-0.8)}{36{,}630\,\pi\,\sqrt{0.8}}$$

$$= 0.00106nm \quad << FSR = 0.0149\ nm \text{ in part b}$$

$$RP = \frac{m\,\pi\,\sqrt{\rho}}{(1-\rho)} = 5.15 \text{ x } 10^5$$

$$F = \frac{RP}{m} = 14$$

17-9. The mirror in a Michelson-type FTS moves at a velocity of 4 mm/sec, and the light incident onto the FTS comes from a helium-neon laser (633 nm).

A. What is the frequency of oscillation of the photocurrent coming from the detectors?

B. By contrast, what is the frequency of the light itself?

SUGGESTED SOLUTION:

A) Frequency of oscillation of photo current

$$f = \frac{2v}{\lambda} = \frac{2\left(4 \text{ x } 10^{-3} \cancel{m} / \text{sec}\right)}{0.633 \text{ x } 10^{-6} \cancel{m}}$$

$$= 12.6 \text{ x } 10^{3} \text{ / sec OR}$$

$$= 12.6 \text{ kHz}$$

B) Frequency of EM wave

$$\text{f} = \frac{\text{c}}{\lambda} = \frac{3 \text{ x } 10^{8} \cancel{m} / \text{sec}}{0.633 \text{ x } 10^{-6} \cancel{m}}$$

$$= 4.74 \text{ x } 10^{14} \text{ Hz}$$

17-11. A hyperspectral sensor utilizes a grating and a 640 element by 480 element array of detectors. It is flown on board an airborne platform flown at 40,000 feet altitude and airspeed of 400 miles per hour. Data is collected in pushbroom mode with a swath width on the ground of 640 pixels and 480 spectral bands.

A. If nearly square pixels, 1 meter in size, are desired on the ground, what line rate must be used (that is, how frequently must the data collected by each row of detectors be sampled and recorded)? Assume that detector dwell time is much less than the sample time.

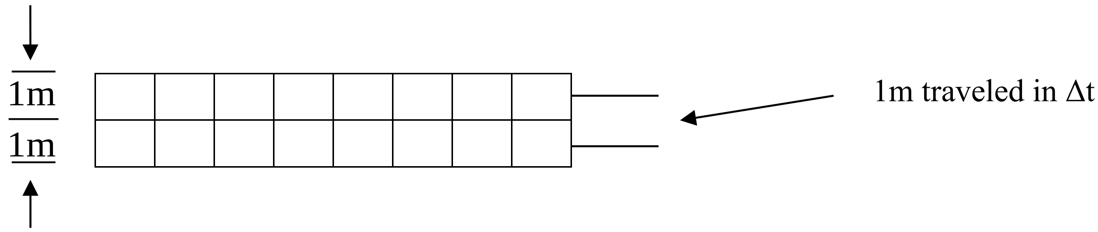
B. How long will it take to collect the complete spectra for a row of pixels on the ground?

C. Suppose the grating is now replaced by a thin wedge-shaped filter placed directly in front of the FPA, how long will it take to collect the complete spectra for a row of pixels on the ground?

D. In either case above, what is the data rate in elements/sec (not bits or bytes)?

SUGGESTED SOLUTION:

A)



$$\Delta t = \frac{1\cancel{m}}{\left(400\,\cancel{mi}/\cancel{hr}\right) \text{ x } \left(5280\,\frac{\cancel{ft}}{\cancel{mi}}\right) \text{ x } \left(\frac{1\,\cancel{hr}}{3600 \text{ sec}}\right)\left(\frac{1\,\cancel{m}}{3.27\,\cancel{ft}}\right)}$$

$$= 5.6 \text{ x } 10^{-3} \text{ sec}$$

$$\text{line rate} = \frac{1}{\Delta t} = \frac{1}{5.6 \text{ x } 10^{-3} \text{ sec}} = 179.4 \text{ lines/sec}$$

B) $5.6 \text{ x } 10^{-3}$ sec because all bands for a pixel are collected at the same time

C) Now the bands are collected sequentially as is done in thin film MS scanners

$$\Delta t = 480\, t_o$$
$$= 480 \left(5.6 \text{ x } 10^{-3} \text{ sec}\right)$$
$$= 2.7 \text{ sec}$$

D) Data rate = (line rate) x (pixels / line) x (#bands / pixel)
= (179.4 lines / sec) (640 / line) (480)
= 55,000,000 data elements / sec